Name and Surname : ………………………………………………………………………...

Grade/Class : 12/………. Mathematics Teacher : ………………………….

Hudson Park High School

GRADE 12
MATHEMATICS
June Paper 1

Marks :

150

Time : 3 hours   Date   : June 2019
Examiner : SLT   Moderator(s) : PHL

INSTRUCTIONS

1. Illegible work, in the opinion of the marker, will earn zero marks.

2. Number your answers clearly and accurately, exactly as they appear on the question paper.

3. **NB** ○ **Start each new Question at the top of a page.**
   ○ **Leave *2 lines* open between each of your answers.**

4. **NB** ○ **Fill in the details requested on the front of this Question Paper.**
   ○ *Do not staple your Question Paper and Answers together.*
     They will be handed in separately.

5. **Detach the Answer Sheet for Question 7 and staple it, in order, with your other answers.**

6. Employ relevant formulae and show all working out. Answers alone may not be awarded full marks.

7. (Non-programmable and non-graphical) Calculators may be used, unless their usage is specifically prohibited.

8. Round off answers to 2 decimal places, where necessary, unless instructed otherwise.

9. If (Euclidean) Geometric statements are made, reasons must be stated appropriately.

QUESTION 1

1.1.   Solve for :

      1.1.1. $3x^2 = 5x$ (3)

      1.1.2. $2x - \dfrac{3}{x} = 7$ (4)

      1.1.3. $3x^{-\frac{2}{5}} = 0{,}81$ (3)

      1.1.4. $x(x - 5) > 6$ (4)

1.2.   Solve for x and y :

      $1 = 2y - x$ and $x^2 - xy + y^2 = 7$ (6)

1.3.   CALCULATORS MAY NOT BE USED IN THIS QUESTION

      1.3.1.   Simplify fully : $\dfrac{3^{2020}}{3^{2014} - 3^{2018}}$ (2)

      1.3.2.   Solve for x : $9^{x+1} + 26.3^x = 3$ (5)

      1.3.3.   If $3^{\sqrt{y}} = 8$, determine the value of $\sqrt[3]{3}^{\sqrt{y}}$ (3)

                                                                  [30]

QUESTION 2

2.1.   How many terms are there in the following series :

$$4 + 1 - 2 - 5 \ \cdots \ = -10\,875$$

(5)

2.2.   If : $\displaystyle\sum_{k=1}^{5} (x - 3k) \ = \ \sum_{k=1}^{8} (x - 3k)$, calculate the value of x.

(3)

2.3.   For a certain quadratic number pattern, the following details are known

- the first three first differences are : $-23 \, ; \, -39 \, ; \, -55$

- the sixtieth term is $-28\,727$

Determine an expression for T_n, the general term of the sequence.

(5)

[13]

QUESTION 3

3.1.   Prove that the sum of the first n-terms of a geometric series is given by

$$S_n = \frac{a(r^n - 1)}{r - 1} \qquad (r \neq 1)$$

(5)

3.2.   Evaluate : $\displaystyle\sum_{k=5}^{22} \frac{3}{4}\left(-\frac{2}{3}\right)^{8 - k}$

(5)

3.3.   Given below are the first three terms of an infinite geometric series

$$(5x + 2) + (2 - 4x) + (x + 7) + \cdots$$

3.3.1.   Calculate the value(s) of x.

(4)

3.3.2.   Now, if the given series converges, calculate the sum to infinity, S_∞.

(4)

3.4.   For a certain geometric series, it is known that

- the sum of the first three terms is 17

- the sum of the sixth, seventh and eighth terms is 544

Calculate the constant ratio, r, of the series.

(4)

[22]

QUESTION 4

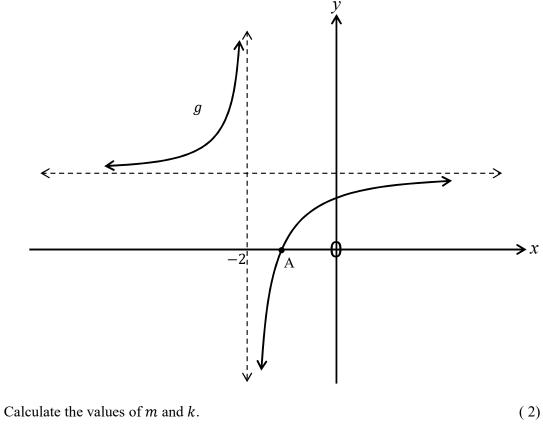4.   Determine an expression for the sum of the first n-terms, S_n, of the following series

$$\frac{1}{4} + \frac{11}{20} + \frac{7}{10} + \frac{11}{14} + \frac{47}{56} + \cdots$$

[2]

QUESTION 5

5.1.   Given : $f(x) = -\dfrac{3}{x+4}$

      5.1.1.   Write down the domain of f. (1)

      5.1.2.   State the equations of the asymptotes of f. (2)

      5.1.3.   Sketch a rough graph of f, showing all relevant details
                on the diagram. (3)

      5.1.4.   If f is reflected in its vertical asymptote to become g, write down the
                equation of g in y-form. (1)

      5.1.5.   Write down the equation of the axis of symmetry of h, if

$$h(x) = f(x) \quad (x > -4)$$ (2)

5.2.   Calculate the coordinates of the reflection of A$(-7; 9)$ in the line $y = -x + 5$. (2)

5.3.   Write $y = \dfrac{3 - 4x}{x + 5}$ in the form $y = \dfrac{k}{x - p} + q$ (2)

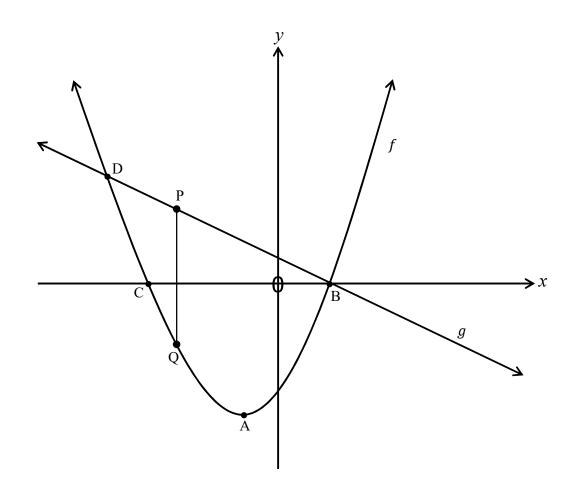5.4.   The asymptotes of $g(x) = \dfrac{3x - m}{x + k}$ are indicated by the short-dashed lines.

The vertical asymptote crosses the x-axis at -2 and A$\left(-\dfrac{2}{3}; 0\right)$.

Calculate the values of m and k. (2)

[15]

QUESTION 6

6.1.   For f, the following details are known

- Axis of symmetry   : $x = -2$
- Range   : $y \in [-18; \infty)$
- A is the turning point of f

The equation of g is $g(x) = -2x + 2$.
PQ is a vertical line whose length is $24\frac{1}{2}$ units.

6.1.1.   Determine the coordinates of

(a)   A   (1)

(b)   B   (1)

(c)   C   (1)

6.1.2.   Now, show that the equation of f will be $y = 2x^2 + 8x - 10$ (4)

6.1.3. Calculate the coordinates of

    (a)   D   (5)

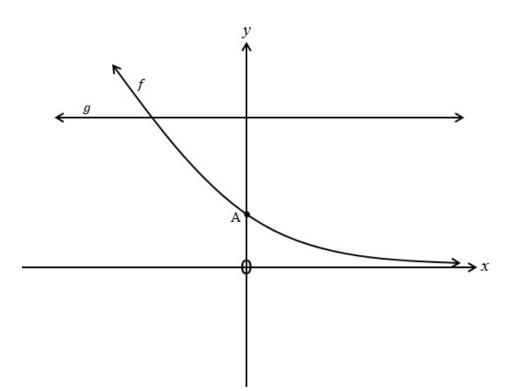   (b)   P   (5)

6.1.4. Use the graphs to solve for x :

    (a) $x.f(x) > 0$ (2)

    (b) $\dfrac{f(x)}{g(x)} \le 0$ (2)

6.2.   Sketch a rough graph of $y = ax^2 + bx + c$ if $a < 0, b > 0, c < 0$
and $b^2 - 4ac = 0$.   (4)

[25]

QUESTION 7

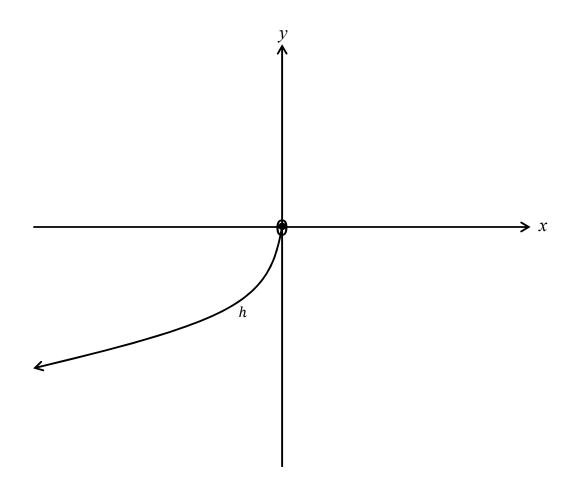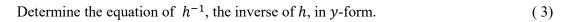**USE THE ANSWER SHEET PROVIDED**

7.1   In the diagram below, $f(x) = \left(\frac{1}{3}\right)^x$ and $g(x) = 3$.

7.1.1.   Write down the coordinates of A.   (1)

7.1.2   On the set of axes, given in the Answer Sheet, sketch the graph of f^{-1},
           the inverse of f.   (2)

7.1.3.   Solve for : $\log_{\frac{1}{3}} x = 3$ (1)

7.1.4.   Hence, write down the solution to : $\log_{\frac{1}{3}} x \geq 3$.   (2)

7.2. The graph of $h(x) = -\sqrt{-3x}$ is shown :

Determine the equation of h^{-1}, the inverse of h, in y-form. (3)

[9]

QUESTION 8

8.1. How many years will it take for a vehicle to depreciate to half of its original value, if the rate of depreciation is 12 % p.a. calculated on the reducing balance method.   (3)

8.2. Convert an effective annual interest rate of 15 % p.a. to a nominal interest rate, as a percentage, p.a. compounded monthly.   (4)

8.3. On the 1st January 2019, a pupil invests R 1 500 in a new savings account that earns interest of 7 % p.a. compounded monthly.
What will be the balance in the account on the 31st December 2030 ?   (4)

[11]

QUESTION 9

9. Given : $f(x) = 30x^3 - 49x^2 + 9x + 4$

9.1. Use the factor theorem to show that $(2x - 1)$ is a factor of f.   (2)
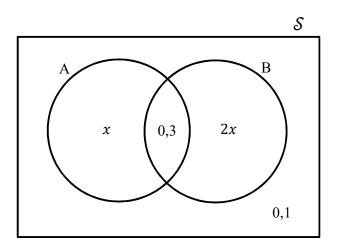
9.2. Hence, factorise f fully.   (3)

[5]

QUESTION 10

10.1.   Two events, A and B, are mutually exclusive. It is also known that

- $P\big((A \cup B)'\big) = 0{,}3$
- $P(A) = 0{,}2$

Calculate $P(B)$. (3)

10.2.   Given below is a Venn Diagram for two Events, A and B :

$\mathcal{S}$

A   B

x $0{,}3$ $2x$

$0{,}1$

10.2.1.   Calculate the value of x, showing that it will be 0,2. (1)

10.2.2.   Are Events A and B independent ? Justify your answer appropriately. (5)

10.3.   In a factory, three machines viz. A, B and C, are used to manufacture glass bottles.
These machines produce 20 %, 30 % and 50 % of the total production, respectively.
Of the glass bottles produced by machines A, B and C, 1 %, 2 % and 6 %, respectively,
are defective.

10.3.1.   Represent the given information in the form of a tree diagram.
Show all relevant details on the diagram. (4)

10.3.2.   A glass bottle is selected, at random, from the total production.
What is the probability that the glass bottle

(a)   was produced by machine B and is not defective ? (2)

(b)   is defective ? (3)

[18]

TOTAL   150